

STUDIES OF HUMAN DYNAMIC SPACE  
ORIENTATION USING TECHNIQUES OF CONTROL THEORY

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## I. INTRODUCTION AND SUMMARY

The Man-Vehicle Control Laboratory has been continuing its study of the problem of dynamic space orientation under the direction of Professors Y. T. Li, Laurence R. Young and Jacob L. Meiry and with the active participation of approximately 15 graduate students in the Department of Aeronautics and Astronautics. We have been considering three major aspects of the problem: models for the human sensors, models for the human operator performance in a control task and application of these models. This report reviews some of this work carried out in the latter half of 1966.

The primary research on sensor models continues to be on the vestibular system and the role of motion cues in pilot performance. A new single axis simulator is being constructed and three experimental programs are being conducted in this area. In addition our older eye-movement model has been revised and simulated on our hybrid computer.

Our major new application effort in this period has been in the development of 3-D display techniques for use with a cathode ray tube. By using kinesthetic cues to get monocular stereo effects, relating the displayed cube to vehicle motion and pilot's head position, we can achieve some degree of depth perception with a two dimensional display. In addition, our work on the use of the phase plane display for optimal control of high order systems has nearly been completed.

A major new approach to adaptive and learning behavior modelling was the result of an application of Bayesian updating to a discrete stochastic learning model. The learning process is modelled as a revision of a priori estimates of an optimum

switching line in state space. The theory attempts to explain how an operator learns to regulate the state of a dynamic system. Another modelling investigation was devoted to the multiloop control problem in which the operator was modelled by a cascade configuration. Using helicopter control as an example the first human operator block controls attitude and the second portion provides the attitude reference. Finally, we completed our work on construction of a rate gyro autopilot for motorbike stabilization, based on the principles of human stabilization.

## II. SENSOR MODELS

### 1. Effects of Motion Cues on Performance

In our continuing study of the vestibular system the effects of motion cues are being systematically examined. Depending on the circumstances, motion cues may be a help or a hindrance to human performance in a man-vehicle system. An extensive study of data taken over a wide range of vehicle dynamics is being made to determine exactly when motion cues are helpful. Hopefully, we will be able to derive from the data a theory which will permit the prediction of moving-base performance from fixed-base data and significantly reduce the time and money involved in moving-base simulation.

A program has been developed and tested which will permit the testing of subjects in fixed or moving base over a wide range of dynamics, and automatically generate as output:

1. the integral square error for a 90 second run,
2. the linear part of the human operator's describing function, amplitude, and phase, and
3. the magnitude of the remnant part of the human operator's describing function.

The testing of this program has consisted of running it with direct feedback, running it through known filters, and taking preliminary data for operator control of fixed base vehicle dynamics of the type  $1/s$ ,  $1/2s-1$ , and  $1/s+1$ . Testing of subjects is expected to begin early in the spring. The tests will be fixed-base at first, and moving-base later, when the NE-2 simulator is prepared. The data will be reduced and tabulated during the experimental period (about four months). The results of this study should be a quantitative generalization of the effects of vehicle motion of pilots' control ability.

## 2. Construction of Single Axis Rotation Device for Vestibular Studies

A rotating chair simulator has been designed and is currently being constructed. The chair portion will be an entirely enclosed box in which the subject will sit upright, seeing no outside reference. His head will be supported by a head rest and his mouth fixed to a bite board. He may wear photoreceptor spectacles to monitor eye nystagmus and he may manipulate a joystick. His head will be positioned above the axis of rotation.

The chair will be rotated about a vertical axis by two  $7\frac{1}{2}$  ft-lb torque motors connected directly to the shaft of the chair. This amount of available torque will allow the chair to be driven with a sinusoidal acceleration whose peak ranges from  $10\text{ degrees/sec}^2$  to  $100\text{ degrees/sec}^2$  at any frequency in the range .01 cps to 1 cps. A single speed resolver is used for closing a position feedback loop. In open loop operation the number of revolutions is unlimited, and the chair can be operated at constant velocity or acceleration. The simulator should be completed in the spring. It will be used to extend the experimental results of Hixson and Niven on nonlinear effects in nystagmus phase and to investigate the thresholds of perception and nystagmus for angular acceleration. It will also be used in the canal caloric experiments; subjects who have previously been involved in caloric tests of their vestibular systems will be given latency time tests using the chair open loop. They will be given a step of acceleration and asked to push the joystick when they feel rotation. The results of these experiments will then be correlated with the results of the caloric tests.

Finally, this rotating chair will be placed on the 32 foot radius centrifuge of the M.I.T. Instrumentation Laboratory for counter-rotating experiments to test some of our fluid dynamic models of cross coupling in the canals.

3. The Labyrinthine Fluids and the Caloric Stimulation Model of Semicircular Canals (Also supported in part by NASA Grant NGR 22-009-156)

Properties of the Labyrinthine Fluids

In the past six months, the equipment for the measurement of the density, coefficient of thermal expansion, viscosity, and thermal coefficient of viscosity has been constructed and calibrated, and samples of cat and human perilymph and human endolymph have been measured. The results are presented in Table 1. The aforementioned properties will be evaluated for cat endolymph as soon as samples are available, and the equipment for the measurement of thermal conductivity is being completed.

Newtonian behavior of Human Endolymph

By operating the microviscometer at two different angles of inclination ( $20^{\circ}$  and  $35^{\circ}$ ) it was found that the viscosities did not vary appreciably, although the terminal velocities of the rolling spheres was nearly doubled. The viscosity was also measured after setting for one day, and again after one week. Within the 2% accuracy limitation of the instrument, no measurable change in viscosity was noted. Thus it appears that endolymph is not a shear thinning fluid, nor does its viscosity change with "setting time;" further, its chemical analysis shows a low protein content that is typical of Newtonian fluids.

Verification of the Caloric Stimulation Model

During the past term an experiment has been designed with the assistance of Dr. A. Weiss at the Massachusetts Eye and Ear Infirmary to measure the latency to onset of ocular nystagmus during caloric stimulation. The data from the first subject is currently being analyzed and it is anticipated that a total of ten subjects will be run within the next month.

Analog and digital computer responses of the proposed model to step inputs of caloric stimulus have been evaluated, and the results have been included in the paper "Properties of the Labyrinthine Fluids and Quantification of the Phenomenon of Caloric Stimulation," which was presented at the Third Annual Symposium on the Role of the Vestibular Organs in Space Exploration at Pensacola, Florida, January 27, 1967, by Steer, Young, Li, and Meiry.

Abstract:

The physical properties of endolymph and perilymph (viscosity, density, thermal coefficient of viscosity, and coefficient of thermal expansion) which are pertinent to the quantification of the dynamic behavior of the human vestibular sensors have been evaluated. Descriptions and error analyses of the instruments used for the measurements are presented.

The phenomenon of caloric stimulation of the semicircular canals is described quantitatively, and a dynamic model is presented. To verify the proposed model, the human's response to caloric stimulation is compared to his response to angular acceleration stimulation.

4. Fluid Dynamic Analysis of Semicircular Canals

The work of the past months in the area of fluid dynamic analysis of a "rigid canal" has led to a system describing function of the form:

$$\frac{\theta_c}{\alpha} (s) = \sum_{i=1}^{\infty} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}$$

where  $\theta_c$  = cupula displacement

$\alpha$  = angular acceleration

where the  $A_i$ ,  $\zeta_i$ , and  $\omega_i$  are function of the viscosity of the

fluid, the coefficients of zero order Bessel functions, and the "spring constant" of the cupula.

In the coming months these coefficients will be determined as accurately as possible from the data available, but the lack of information on the elastic properties of the cupula makes numerical evaluation of  $\omega_1$  very difficult; we will probably have to use data from motion experiments. It is expected that the analysis of a "flexible canal" structure under the influence of counter-rotation will shortly be completed by finding the magnitude of the circulation function as a function of canal flexibility.

## 5. Otolith Models

Based on further consideration of the nature of the otolith response mechanism we have revised our dynamic model of the sensor. This model agrees with all known test data on perception of tilt or linear acceleration and eye counterrolling data. The model is described in a paper submitted to the Third Symposium on the Vestibular Organs in Space Exploration, Pensacola, Florida, January 1967. The abstract of the paper by Young and Meiry is given below.

### Abstract:

The basic dynamic otolith model of Meiry was based on observed relations between perceived direction of linear motion and input acceleration. Although this model correctly predicted phase of perceived velocity for lateral oscillation and time to detect motion under constant acceleration, it failed to account for at least two observations:

- 1 ) Behavioral and electrophysiological data indicate a sustained steady otolith output to sustained tilt angle. The model's perceived acceleration or tilt output decayed to zero with a time constant of 10 seconds.



ii) Dynamic counterrolling data agree with the model at higher frequencies. The experimental counterrolling at zero frequency, however, indicates a static component of otolith output with no phase lag referred to acceleration; whereas the model had no static output and approached 90 degrees of lead at zero frequency.

At the suggestion of Dr. H. Von Gierke, a static component was included in the otolith model. The revised linear model allows steady state response to acceleration.

This revised linear model will act approximately as a velocity transducer over the mid-frequency range ( $0.19 < \omega < 1.5$  rad/sec.). The transfer function from specific force to perceived tilt or lateral acceleration has a static sensitivity of 0.4.

## 6. Eye Movement Model

The sampled data model for eye tracking movements developed by Professor Young is being simulated on the hybrid computer, and several modifications are being incorporated. The most important is the postulation of a continuous pursuit loop acting on target rate. Additionally, running the simulation at a variety of possible synchronizations between input changes and sampling instants (as postulated but not pursued in the original model) reveals all the different types of eye movement transient response observed and reported in the literature.

### III. DISPLAYS

#### 1. The 3-D CRT Display

Since September, 1966, the Laboratory has directed a substantial portion of its energy to the development of a computer generated visual display in which various "depth cues" are included in the presentation on a two dimensional CRT screen.

The first step was to discover what parameters are important in the perception of depth. By performing the normal psychophysical experiments with different size cubes and spheres held motionless in a dark room it was verified that

1. Binocular vision is a very strong depth cue.
2. Monocular vision with the absence of head movement gives no depth cue.
3. Monocular vision with head movement gives almost as strong a depth cue as binocular vision.

These experiments were performed with the objects no more than fifteen feet away from the viewer. It was concluded that head movement provides a strong cue to the change in depth of the object being viewed when the object is at close range. Therefore, the 3-D display will eventually include some type of head position sensor so that the 3-D representation on the CRT screen will change in perspective as the head is moved.

A cube was chosen as the initial object to be displayed. Preliminary work produced a static display. The ultimate goal is to allow the observer a full six degrees of freedom of apparent motion about the cube; the changing aspect produces a depth cue similar to that of head motion.

Our hybrid computer was used for the display generation, with the visual output presented on a dual beam oscilloscope. The digital portion of the computer is used to generate and store the display list of lines to be shown. The list is converted line by line from digital to analog on request by the analog

computer, which creates the actual voltages used to drive the CRT sweep. These data transfers are done through the use of interrupt programming so that the display list calculations continue when the digital portion of the machine is not engaged in the transfer of data. The cube lends itself well to being drawn with its edges as four triads of lines. This approach required fewer data transfers and allowed a very symmetrical logic. The information transmitted is the screen coordinates of the vertex of each triad and the slope of each line associated with that vertex.

The analog computer uses the vertex coordinates as the initial condition for three integrations. The integrations are of  $\Delta x$  and  $\Delta y$  of each line for a period  $T$  with a gain  $1/T$ . This series of ramps provides the horizontal and vertical deflection signals to the oscilloscope and are parametric in time and synchronized. The oscilloscope has an input for electrical control of the beam intensity, which is used to blank the trace as it moves between lines.

Under test it was demonstrated that the analog integrators could accurately generate the synchronized deflection signals for the oscilloscope from data provided by the digital computer. This technique at present can display 500 lines per second. At 30 frames per second, the display is limited to 16 lines to describe the object or objects.

The program to allow the observer a full six degrees of freedom takes a major portion of the digital computer time. Essentially, the approach is to keep track of the three dimensional coordinates of the cube vertices in body axes (Fig. 1), referenced to the observer's eye, and to continually update these coordinates as the observer translates and rotates in space. Perspective equations (Fig. 2) are then used to calculate the two dimensional coordinates of the vertices on the display screen, allowing for observer head movement with relation to the CRT. These coordinates are then rearranged into the point and slope format of the triad display list.

Under a test, a moving cube was displayed in the proper perspective. The signals of a pilot control stick can now be converted from analog to digital for use in the coordinate updating calculations. In this way, direct two dimensional translational position control has been obtained.

Scene rotation of the three dimensional coordinates required careful review of mathematical techniques: If the three space coordinates of each vertex point are operated upon successively by an orthogonal coordinate transformation matrix of the type shown in Figure 3, any arbitrary rotation in space can be reduced to the resultant of a single pitch, roll, and/or yaw maneuver using the product of these rotation matrices. Unfortunately, matrix multiplication is not, in general, commutative, so that the magnitude of the roll, pitch, and yaw angles necessary to achieve a given arbitrary rotation depends very much on the order in which they are performed. For the purposes of the display calculations, a given rotation would have to be built up out of a series of small pitch, roll, and yaw transformations, since the matrix product of these transformations does commute to the first order. Thus, using small angular increments, a reasonable approximation to a given rotation can be achieved by a series of nearly simultaneous rotations through small angular increments about the observer's coordinate axes.

Since a "table look up" technique could not be conveniently used, it was hoped that a first order approximation for the sine and cosine terms in the transformation shown in Figure 3 could be used. A program developed using this transformation (Figure 4) showed a large, unstable distortion of the coordinate field after only a few rotations. Investigation showed that, indeed, the Eigenvalues of this transformation are complex, and have a magnitude greater than one. However, a transformation similar to that shown in Figure 5 should allow stable rotations to be performed, since the Eigenvalues of this transformation, though complex, do have unity magnitude. Computer implementation of

this transformation as a series of difference equations appears simple and promising.

In the coming months our objectives are to refine the accuracy and increase the speed of the three dimensional display and to develop a head position sensor. With greater computational speed, more than one object can be shown on the display. Currently, compensation is made for line length so that long lines have the same brightness as short lines, and we are incorporating blanking and brightness control inversely proportional to the distance of the points behind the screen. This will serve as an additional depth cue. In its completed form, the display system will be expanded to the full six degrees of freedom plus one degree of pilot head motion. Tests will be run to assess the effect of the depth cues on a pilot's ability to perform various control tasks with simulated vehicle dynamics.

## 2. Switching Locus Phase Plane Display

Continuation of our work on the usefulness of phase plane displays for control of high order systems led to the current work on "near-optimal" manual control. The addition of a time optimum switching line to the display has been shown to improve manual control. In this study the concept was extended to the generation of a moving "switching locus" on the phase plane for control of high order systems. The results, while very encouraging, do not point clearly to an extension of the technique to fourth order systems because of computational complexity. The results of this work were summarized in the paper "Manual Time Optimal Control for High Order Plants" by Yasui and Young, submitted for presentation at the USC-NASA Conference on Manual Control, March 1967. The abstract follows:

### Abstract:

The concept of a switching surface (SS) is applied to closed-loop manual time optimal bang-bang control of high order systems by visually displaying the

instantaneous position-velocity error spot and the switching locus (SL) simultaneously on the phase plane. SL is a curve describing the intersection between SS and the plane parallel to the phase plane; for example representing the locus in the instantaneous acceleration plane for third order plants. The operator's task is to achieve minimum time response by reversing control polarity when the moving state point intersects the moving SL.

The general procedure is outlined for n-th order plants with real poles, and a technique suitable for an analog computer to generate the exact or approximate SL is proposed. The second order plants,  $1/s^2$  and  $1/s(s + \alpha)$ , are presented as preliminary examples, for which the switching loci do not move. The third order plant,  $1/s^3$ , is studied in detail as an example of higher order plants. Satisfactory experimental results have been obtained for all examples given.

#### IV. HUMAN OPERATOR MODELLING

Two major projects in modelling the human operator were completed during recent months. These modelling efforts both represent our current pattern of departing from conventional quasi-linear modelling, which is often sufficiently accurate for engineering purposes, and concentrating on some of the "fine structure" which may indicate the organization of the operator's control.

##### 1. Learning Behavior

A new approach to adaptive and learning behavior in manual control was presented by A. E. Preyss in his Sc.D. thesis (MVT-67-1) the abstract of which follows:

##### Abstract:

A theory is presented for the explanation of human learning behavior in a manual control task. In the performance of a psychomotor task, a human operator responds to sensory stimuli with limb movements. This complex psycho-physiological phenomenon is conceptualized as a single channel information processing system. A sensor, a decision center and an effector are the serially connected components of the system. Transmission and processing of information take time, and the delay between the reception of a sensory stimulus and the execution of a motor response is treated as a finite sum of component times which are assumed to be statistically independent random variables.

In the decision center responses are selected from a set of possible control policies. Stored in memory are apriori estimates of the probability that a specific policy should be in force at the moment of decision. Policy selection is determined by a rule which takes the priors into account. Learning is effected by a revision of the priors based on the weighting of certain evidence. Bayes' theorem is the revision rule. Simple and readily perceived events in the state history of the dynamic process being controlled are used for evidence in resolving control policy uncertainty.

A model of human learning behavior is the computer program obtained from a translation of the theory into machine language. Behavior of the model depends not only on the rules of information processing postulated by the theory, but also on a set of parameters characterizing the mental and physical attributes of an individual. Model behavior is compared with subject behavior observed in a motor skill experiment performed at M.I.T.'s Man Vehicle Laboratory.

As set forth, the theory explains how a human operator learns to regulate the state of a dynamic process using a relay controller. Generalization of the theory to other task contexts is discussed.

## 2. Multiloop Control

An approach to the multi-loop manual control problem has been taken assuming a cascade model for the operator. The use of average transient responses to ramp inputs as a rapid identification technique facilitated study of the situation. The investigation is described in report MVT-67-2 "Helicopter Control: A Multi-Loop Manual Control System," by G. R. Friedman.

### Abstract:

The human operator's performance in a high order, multi-loop task, typified by the helicopter, is studied using the method of average responses. This method permits a time domain, transient input analysis. A cascade model configuration for the human operator is proposed. In this configuration, the first human operator model controls attitude. This model is identical to that of the single-loop model for the same dynamics and consists of a lead time constant of 5 seconds, a neuromuscular lag of .1 second, and a pure time delay of .28 seconds. The attitude reference for the attitude control loop is provided by a second cascade human operator model consisting of a one second lead operating on the position error. A general programming system for average response experiments, using the GPS 290T Hybrid Computer, is described.



### 3. A Rate Gyro Autopilot for a Motorbike

In riding a motorbike the human operator makes use of visual and vestibular information to control both roll and heading. Our study of the human behavior on this vehicle led to the design of an autopilot to perform the task. The most recent effort in this direction was the S.M. thesis of Lewis M. Nashner, entitled "A Rate Gyro Autopilot for a Motorbike," M.I.T., Course XVI, January 1967, the abstract of which follows.

#### Abstract:

The object of this thesis is to develop an autopilot which maintains the roll stability of a motorbike and guides it on a course commanded by a passive rider on the motorbike. The single control parameter of the autopilot is the steering wheel angle. The feedback sensor is chosen to give dynamic information closely related to that which is available to a human rider, the sensory outputs of the vestibular system.

The two most appropriate sensors, the pendulum and the rate gyro, are initially considered in simplified linear studies. The rate gyro sensor is found to give superior performance and is chosen as the feedback sensor.

The autopilot configuration is first simulated on an analog computer to accurately determine the requirements of the drive, the feedback compensation, and the loop gain. The actual hardware is assembled to conform to the specifications of the simulated autopilot.

The autopiloted motorbike has a well damped response to step torque disturbances resulting in lean angles of up to 30 degrees. The response limitation of the autopilot is primarily due to its non-adaptive loop gain. While a single feedback compensation for all speeds is used, a specific setting of feedback gain limits the range of bike speeds for which the autopilot is stable to a small region.

The autopiloted motorbike is found to be directionally unstable when it is commanded to maintain a straight course. Analysis shows that considerable improvement in the accuracy and elimination of drift in the roll angle sensor will not decrease the directional instability significantly. Feedback information of the lateral error of the bike on its track is necessary if the system is to maintain a straight course within a 15 meter tolerance for distances over 200 meters.

## V. PUBLICATIONS

Li, Y. T., and J. L. Meiry, "An Active Roll Mode Suspension System for Ground Vehicles," presented at JACC, August 1966, accepted for publication in ASME Transactions.

Young, L. R., "The Dead Zone to Saccadic Eye Movements," June 1966, Symposium on Biomedical Engineering, Marquette University (Charles C. Thomas, in press).

Li, Y. T., "Design Optimization of a Multimode Hydraulic Vehicle Suspension System," presented in AGARD Lecture Series, September 1966, Turin, Brussels, London.

Young, L. R., and R. Winblade, "MIT-NASA Working Conference on Manual Control," IEEE Spectrum, Nov. 1966, pp. 88-93.

Steer, R. W., Y. T. Li, L. R. Young, and J. L. Meiry, "Physical Properties of the Labyrinthine Fluids and Quantification of the Phenomenon of Caloric Stimulation," Third Symposium on the Role of the Vestibular Organs in Space Exploration, Pensacola, Florida, January 23-27, 1967.

Young, L. R., and J. L. Meiry, "A Revised Dynamic Otolith Model," Third Symposium on the Role of the Vestibular Organs in Space Exploration, Pensacola, Florida, January 23-27, 1967.

Yasui, S., and L. R. Young, "Manual Time Optimal Control for High Order Plants," presented at the USC-NASA Conference on Manual Control, March 1967.

### Theses

Nashner, L. M., "A Rate Gyro Autopilot for a Motorbike," S.M. Thesis, M.I.T., January 1967.

Preyss, A. E., "A Theory and Model of Human Learning Behavior in a Manual Control Task, Sc.D. Thesis, M.I.T., February 1967.

Friedman, G. R., "Helicopter Control: A Multi-Loop Manual Control System," S.M. Thesis, M.I.T., June 1967.

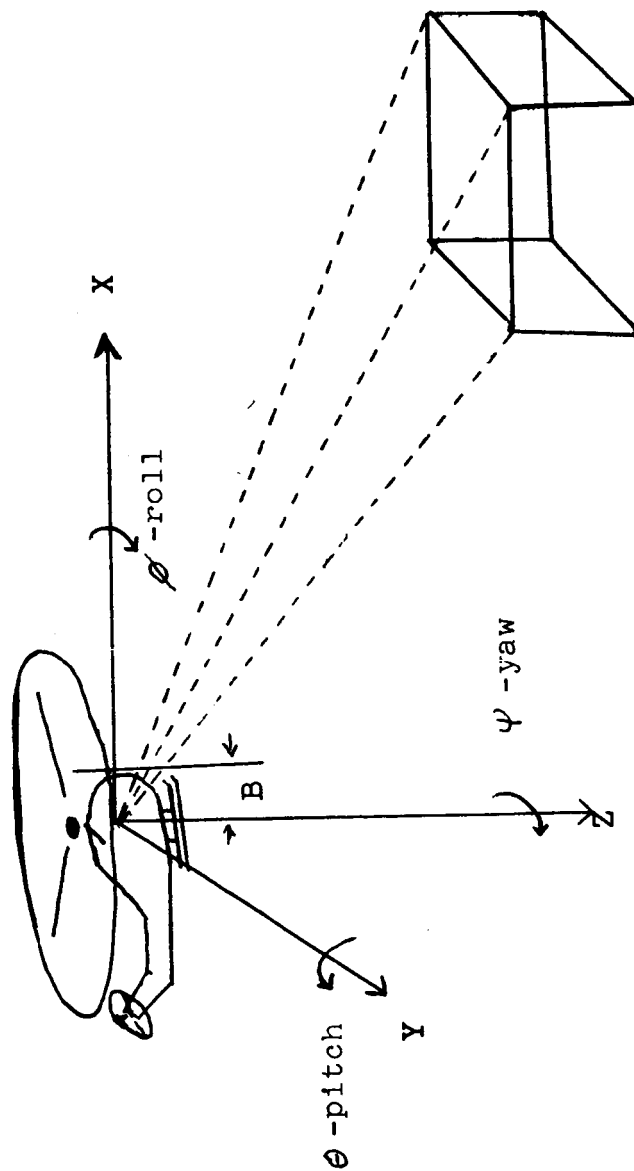
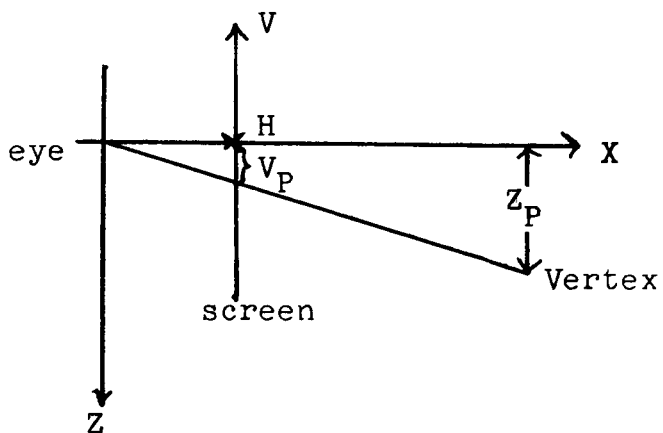


Figure 1. Coordinate system notation. Translation of the coordinate axes controls the location of the observer's eye, while rotation of the axes indicates the orientation of the observer's window.

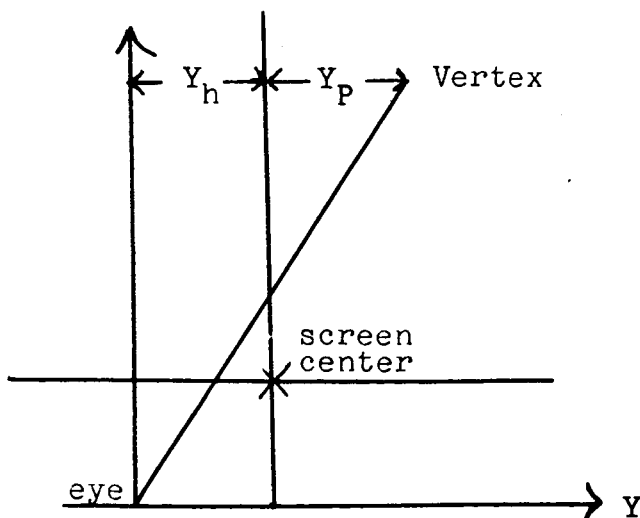
H, V screen axes--displaced from X, Y, Z axes  
by  $Y_h$  along Y axis and eye-screen distance B  
along X axis.

Vertical



$$V_P = - \frac{B}{X_P} Z_P$$

Horizontal



$$H_P = \frac{B}{X_P} (Y_P + Y_h) - Y_h$$

allows for screen  
movement with  
respect to eye.

Figure 2. Perspective Equations

$$\begin{bmatrix} X_P \text{ new} \\ Y_P \text{ new} \\ Z_P \text{ new} \end{bmatrix} = \begin{bmatrix} \cos \Psi & \sin \Psi & 0 \\ -\sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P \text{ old} \\ Y_P \text{ old} \\ Z_P \text{ old} \end{bmatrix}$$

Figure 3. Orthogonal Coordinate Transformation Matrix for yaw rotation. Analogous transformations can be derived for pitch and roll.

$$\begin{bmatrix} X_P \text{ new} \\ Y_P \text{ new} \\ Z_P \text{ new} \end{bmatrix} = \begin{bmatrix} 1 & d\Psi & 0 \\ -d\Psi & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P \text{ old} \\ Y_P \text{ old} \\ Z_P \text{ old} \end{bmatrix}$$

Figure 4. First Order Approximations to Orthogonal Transformation for yaw rotation

$$\begin{bmatrix} X_P \text{ new} \\ Y_P \text{ new} \\ Z_P \text{ new} \end{bmatrix} = \begin{bmatrix} 1 & \frac{d\Psi}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{d\Psi}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{d\Psi}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{d\Psi}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P \text{ old} \\ Y_P \text{ old} \\ Z_P \text{ old} \end{bmatrix}$$

Figure 5. Coordinate Transformations Alternating Order Serial Updating